M: 2

Numericals.

1) Calculate the Value of $\int_0^l L_1 L_2 \, dx$

Soln

We. know

$$\int_A (L_1)^{\alpha} (L_2)^{\beta} dx = \frac{\alpha! \beta!}{(\alpha+\beta+1)!} \times l$$

Comparin $\alpha = 1$, $\beta = 1$

$$\frac{\alpha! \beta!}{(\alpha+\beta+1)!} l = \frac{1! 1!}{3!} l = \frac{L}{6}$$

② Calculat the Valu of $\int_A L_1 L_2^2 L_3^3 dA$

Soln

$$\int_A (L_1)^{\alpha} (L_2)^{\beta} (L_3)^{\gamma} dA = \frac{\alpha! \beta! \gamma!}{(\alpha+\beta+\gamma+2)!} 2A.$$

$\alpha = 1$, $\beta = 2$, $\gamma = 3$.

$$= \frac{1! \times 2! \times 3!}{(1+2+3+2)!} 2A$$

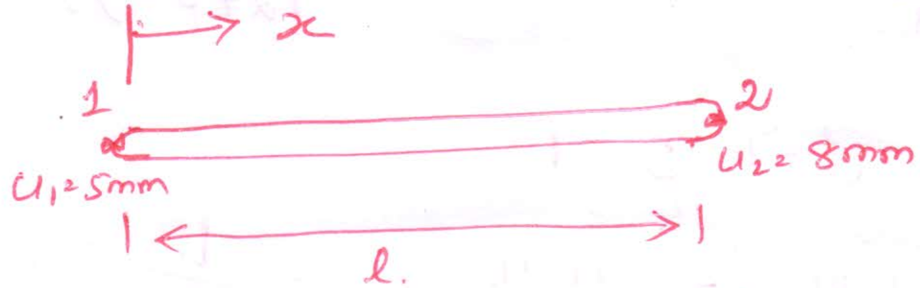
$$= \frac{A}{1680}.$$

AW

① Determine the value of $\int_0^l L_1^3 \cdot dx$.

②

③ A two noded truss element is shown in figure. The nodal displacement are $u_1 = 5mm$ & $u_2 = 8mm$. Calculate the displacement at $x = l/4$, $l/3$ & $l/2$



We know that: $u_1 = 5mm$, $u_2 = 8mm$

$u = N_1 u_1 + N_2 u_2$, $N_1 = \frac{l-x}{l}$, $N_2 = \frac{x}{l}$.

$$u = \left(\frac{l-x}{l}\right) u_1 + \left(\frac{x}{l}\right) u_2 \rightarrow (1)$$

to find displacement at $x = l/4$

$$u = \left(\frac{l - l/4}{l}\right) 5 + \left(\frac{l/4}{l}\right) 8mm$$

@ $x = l/4$, $u = \cancel{6mm}$ $5.75mm$

to find disp @ $x = l/3$

$$u = \left(\frac{l - l/3}{l}\right) 5 + \left(\frac{l/3}{l}\right) 8$$

$u = 6mm$ @ $x = l/3$.

@ $x = l/2$

$$u = \left(\frac{l - l/2}{l}\right) u_1 + \cancel{8} + \left(\frac{l/2}{l}\right) u_2$$

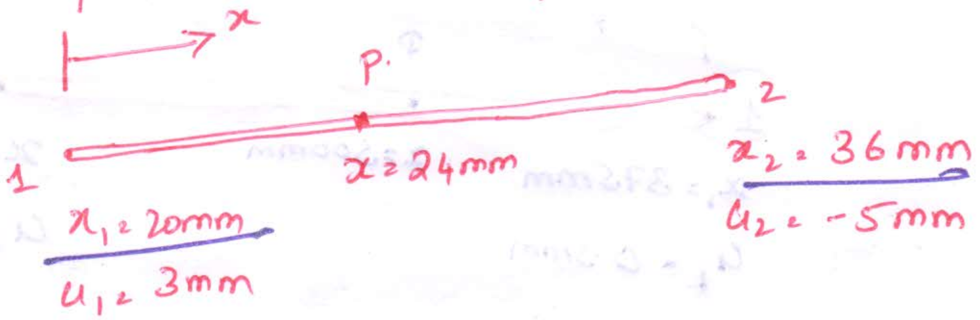
$u = 6.5\,mm$ @ $x = l/2$

---

4. A 1D bar is shown. Calculate the following

(i) Shape function N_1 & N_2 at point P.

(ii) if $u_1 = 3\,mm$ & $u_2 = -5\,mm$. Calculate the displacement u at point P.



Soln

W.K.T. $l = x_2 - x_1 = 16\,mm$

the distance b/n ~~point~~ node 1 and point P is 4mm.

($x = 24 - 20\,mm$)

$u = N_1 u_1 + N_2 u_2$.

$$N_1 = \frac{l - x}{l} = \frac{16 - 4}{16} = 0.75\,mm$$

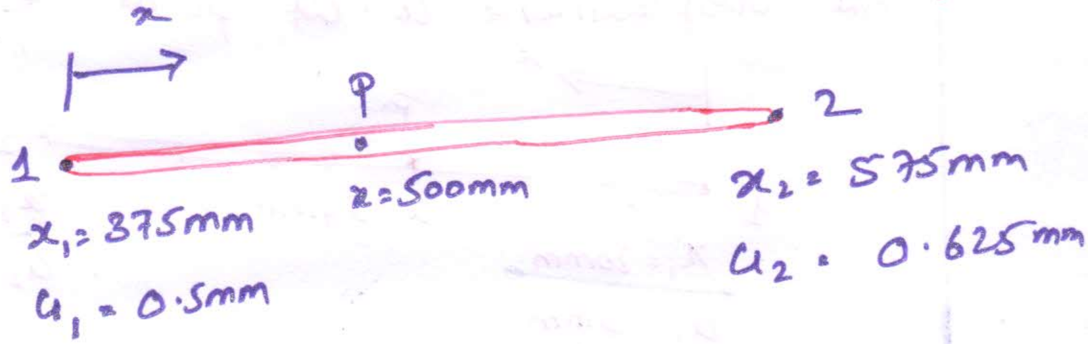
$$N_2 = \frac{x}{l} = 4/16 = 0.25\,mm$$

$u = 0.75 \times 3 + 0.25(-5)$

$u = 1\,mm$

(5) Consider a bar as shown in figure. Cross sectional area of the bar is $750\,mm^2$ and Young's modulus is $2\times10^5\,N/mm^2$. If $u_1 = 0.5\,mm$ and $u_2 = 0.625\,mm$. Calculate the following.

(i) Displacement at point, P. (ii) Strain, e (iii) Stress, σ (iv) Element Stiffness matrix [k] (v) Strain energy, U



Soln:

Length of bar $= x_2 - x_1 = 575 - 375$
$= 200\,mm.$

Distance of P from node 1, $x = 500 - 375$
$= 125\,mm.$

Displacement $u = N_1 u_1 + N_2 u_2$

$$N_1 = \frac{l-x}{l} ; \quad N_1 = \frac{200-125}{200} = 0.375$$

$$N_2 = \frac{x}{l} = \frac{125}{200} = 0.625$$

(i) Displacement

$$u = N_1 u_1 + N_2 u_2$$

$$= 0.375\,(0.5) + 0.625\,(0.625)$$

$$u = 0.5781 \text{ mm}$$

(ii) Strain(e) $= [B]\{u^*\}$

$$[B] = \begin{bmatrix} -\frac{1}{l} & \frac{1}{l} \end{bmatrix}, \quad \{u^*\} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{200}, & \frac{1}{200} \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0.625 \end{Bmatrix}$$

$$e = 6.25 \times 10^{-4}$$

(iii) Stress, $\sigma = E \times e$

$$= 2 \times 10^5 \times 6.25 \times 10^{-4}$$

$$= 125 \text{ N/mm}^2$$

(iv) Element Stiffness matrix $[K]$

$$[K] = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{750 \times 2 \times 10^5}{200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K] = 7.5 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

(5) Strain energy $U = \frac{1}{2}\{u^*\}^T [K] \{u^*\}$

$= \frac{1}{2} [u_1, u_2) \times 7.5 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

$= \frac{1}{2} [0.5, 0.625] \times 7.5 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0.625 \end{Bmatrix}$

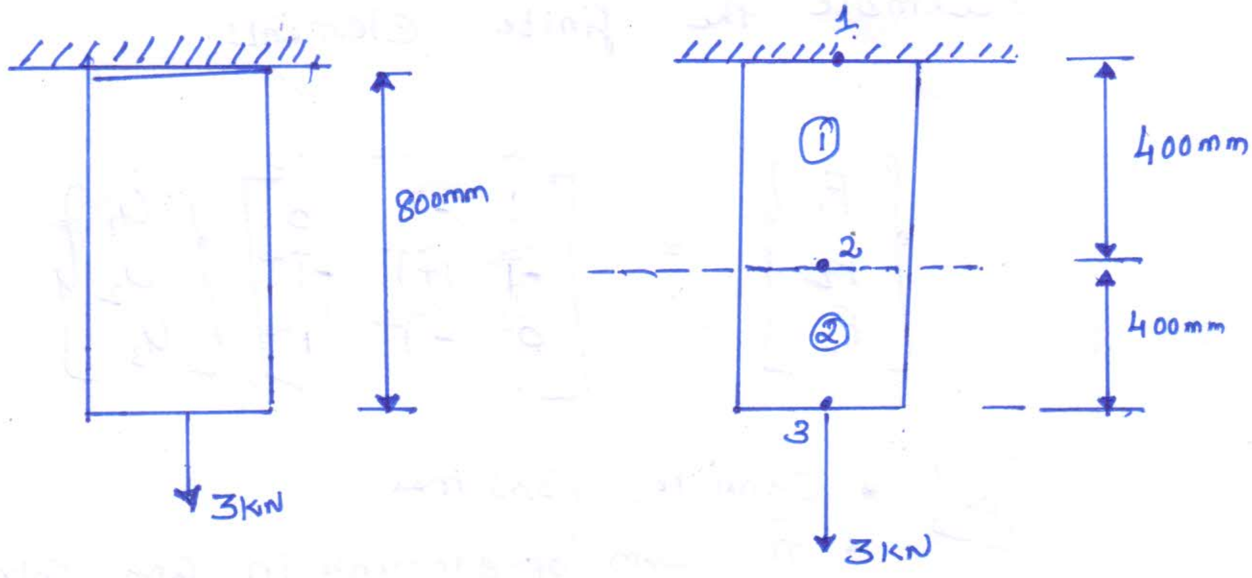
$= \frac{7.5 \times 10^5}{2} [0.5, 0.625] \begin{bmatrix} 0.5 - 0.625 \\ -0.5 + 0.625 \end{bmatrix}$

$= \frac{7.5 \times 10^5}{2} [0.5, 0.625) \begin{Bmatrix} -0.125 \\ 0.625 \end{Bmatrix}$

$U = 5859.37$ N-mm

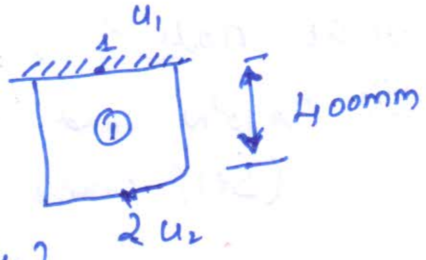
(6) A Steel bar of length 800 mm is Subjected to an axial load of 3 kN as Shown in figure. Find the elongation of the bar, neglect self weight. Take $E = 2 \times 10^5$ N/mm², $A = 300$ mm²

Soln



For element 1 (nodes 1,2)

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

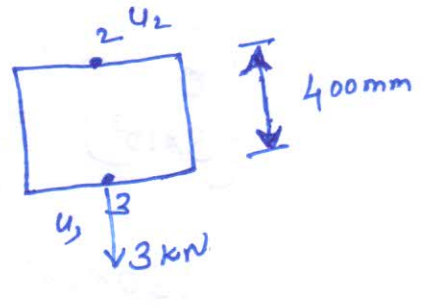


$$= \frac{300 \times 2 \times 10^5}{400} \begin{array}{cc} a_{11} & a_{12} \end{array} \begin{bmatrix} \overset{}{1} & -1 \\ \overset{a_{21}}{-1} & \overset{a_{22}}{1} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= 150 \times 10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element 2 [2,3]

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{300 \times 2 \times 10^5}{400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$



$$= 150 \times 10^3 \begin{array}{cc} a_{22} & a_{23} \end{array} \begin{bmatrix} 1 & -1 \\ \overset{a_{32}}{-1} & \overset{a_{33}}{1} \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

Assemble the finite elements.

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} \overset{a_{11}}{1} & \overset{a_{12}}{-1} & \overset{a_{13}}{0} \\ \overset{a_{21}}{-1} & \overset{a_{22}}{1+1} & \overset{a_{23}}{-1} \\ \overset{a_{31}}{0} & \overset{a_{32}}{-1} & \overset{a_{33}}{1} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad 150\times10^3$$

$[K]$ * Symmetric, 3×3. man.

* The sum of elements in any column are Sahyl.

Apply Boundary Condition.

(i) at node 1, $u_1 = 0$

(ii) 3×10^3N load is acting at node 3, $F_3 = 3\times10^3$N.
(self weight is neglected)

Now sub, val of u_1, F_1, F_2, F_3 in above eqn.

$$\begin{Bmatrix} 0 \\ 0 \\ 3\times10^3 \end{Bmatrix} = 150\times10^3 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{bmatrix} 0 \\ 3\times10^3 \end{bmatrix} = 150\times10^3 \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$\left.\begin{aligned} 150\times10^3\,(2u_2 - u_3) &= 0. \\ 150\times10^3\,(-u_2 + u_3) &= 3\times10^3. \end{aligned}\right\} \text{ Resolving the both eqn weg.}$$

$u_2 = 0.02$ mm, $u_3 = 0.04$ m.m, $u_1 = 0$.

(... elimination method)

HW 2 Solve the ~~Solv~~ previous Numerical by dividing the bar into

(i) 3 equal parts. (Students with even reg.no)

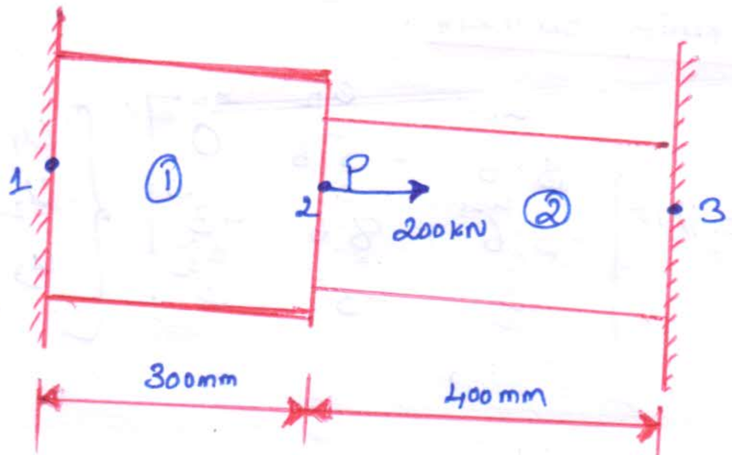
(ii) 4 equal parts (Students with odd. reg.no).

(7) Consider a bar as shown in figure. An axial load of 200kN is applied at point P. Take $A_1 = 2400\ mm^2$, $E_1 = 70\times10^9\ N/m^2$, $A_2 = 600\ mm^2$, $E_2 = 200\times10^9\ N/m^2$. Calculate the following:

(i) The nodal displacement at point P.

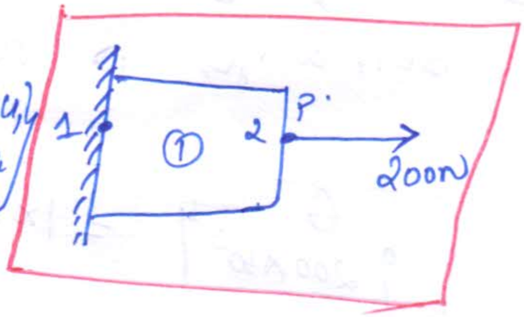
(ii) Stress in each material

(iii) Reaction force.



Element 1 (1 & 2 nodes)

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{2400\times 70\times10^9}{300}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

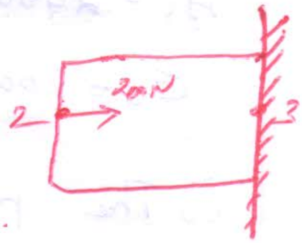


$$= 5.6\times10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= 1\times10^5 \begin{bmatrix} \overset{a_{11}}{5.6} & \overset{a_{12}}{-5.6} \\ \overset{a_{21}}{-5.6} & \overset{a_{22}}{5.6} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Element: 2 (node 2,3)

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{600\times200\times10^3}{400} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$



$$= 3\times10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$= 1\times10^5 \begin{bmatrix} \overset{a_{22}}{3} & \overset{a_{23}}{-3} \\ \overset{a_{32}}{-3} & \overset{a_{33}}{3} \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

Assemble: the finite element:

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} \overset{a_{11}}{5.6} & \overset{a_{12}}{-5.6} & \overset{a_{13}}{0} \\ \overset{a_{21}}{-5.6} & \overset{a_{22}}{8.6} & \overset{a_{23}}{-3} \\ \overset{a_{31}}{0} & \overset{a_{32}}{-3} & \overset{a_{33}}{3} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

Apply boundary Conditions:

(1) $u_1 = 0$, $u_3 = 0$, $F_1 = 0$, $F_3 = 0$, $F_2 = 200\times10^3\,N$.

self weight is neglected.

$$\begin{Bmatrix} 0 \\ 200\times10^3 \\ 0 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.8 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix}$$

$u_2 \Rightarrow 200 \times 10^3 = 1 \times 10^5 \times 8.6 \times u_2$

$u_2 = 0.2325\ mm.$

$\sigma = E \cdot e$
$e = \frac{du}{dx} = \frac{x_2 - x_1}{l}$

(ii) Stress $\sigma = E \cdot \frac{du}{dx}$

$\sigma_1 = E_1 \times \frac{u_2 - u_1}{l_1} = 70 \times 10^3 \frac{(0.2325 - 0)}{300}$

$\sigma_1 = 54.25\ N/mm^2$ (tensile)

$\sigma_2 = E_2 \times \frac{u_3 - u_2}{l_2} = 200 \times 10^3 \times \left(\frac{0 - 0.2325}{400} \right)$

$\sigma_2 = -116.25\ N/mm^2$ (compression)

(iii) Reaction force, $\{R\} = [K]\{u^*\} - \{F\}$

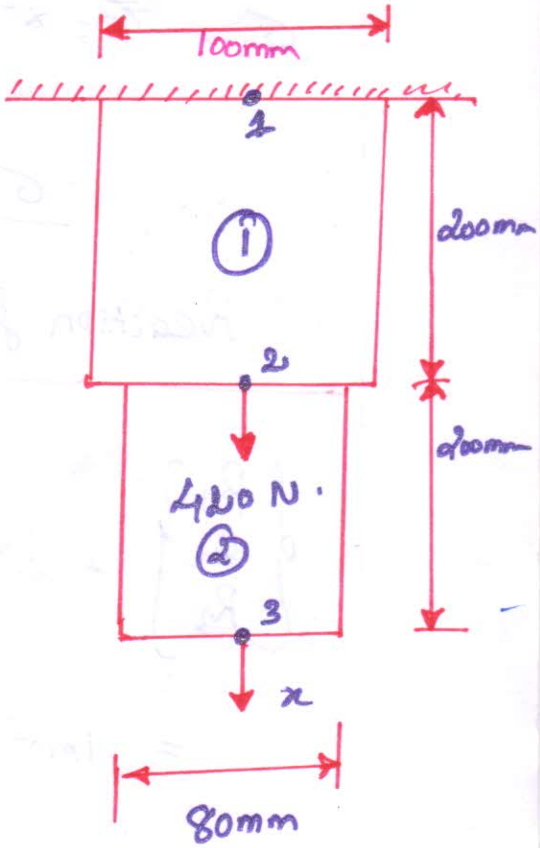
$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$$= 1 \times 10^5 \begin{bmatrix} 5.6 \times 0 - 5.6 \times 0.2325 + 0 \\ 0 + 8.6 \times 0.2325 + 0 \\ 0 - 3 \times 0.2325 + 0 \end{bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -1.302 \times 10^5 \\ 2 \times 10^5 \\ -0.6975 \times 10^5 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -1.302 \times 10^5 \\ 0 \\ -0.6975 \times 10^5 \end{Bmatrix}$$

$R_1 = -1.302 \times 10^5$, $R_2 = 0$, $R_3 = -0.6975 \times 10^5$

8) A thin steel plate of uniform thickness 25mm is subjected to a point load of 420N at mid depth as shown in figure. The plate is also subjected to self weight. If Young's modulus, $E = 2 \times 10^5 \, N/mm^2$ and unit weight density, $\rho = 0.8 \times 10^{-4} \, N/mm^3$, Calculate the following. (i) Displacement at Each Node.



$A_1 = 100 \times 25mm = 2500 \, mm^2$

$A_2 = 80 \times 25mm = 2000 \, mm^2$

Soln:

Body force $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\rho A l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

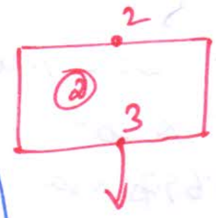
for element 1.

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\rho_1 A_1 l_1}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \frac{0.8 \times 10^{-4} \times 2500 \times 200}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \begin{Bmatrix} 20 \\ 20 \end{Bmatrix}$$

For element 2:



$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{0.8 \times 10^{-4} \times 2000 \times 200}{20} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \begin{Bmatrix} 16 \\ 16 \end{Bmatrix}$$

Assembly the ~~Body~~ Self weight ~~Load~~ ~~force~~, vector.

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 20+16 \\ 16 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 36 \\ 16 \end{Bmatrix}$$

finding the global force vector. ~~force~~

eqn Force at $N_1 = 0, = N_3$, but $N_2 = 420\,N$

→ Self wt   ↙ Point load.

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 36 \\ 16 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 420 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 456 \\ 16 \end{Bmatrix}$$

Stiffness eqn:

for element 1

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = 2\times 10^5 \begin{bmatrix} 12.5 & -12.5 \\ -12.5 & 12.5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

for element 2

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = 2\times 10^5 \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

assembly for finite element

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} 12.5 & -12.5 & 0 \\ -12.5 & 22.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

...ing the boundary conditions. $u_1 = 0$;

$$\begin{Bmatrix} 20 \\ 456 \\ 16 \end{Bmatrix} = 2\times10^5 \begin{bmatrix} 12.5 & -12.5 & 0 \\ -12.5 & 22.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\left.\begin{aligned} 2\times10^5 (22.5u_2 - 10u_3) &= 456 \\ 2\times10^5 (-10u_2 + 10u_3) &= 16 \end{aligned}\right\} \text{ solving the eqn we get}$$

$u_1 = 0.$

$u_2 = 1.888 \times 10^{-4}$ mm

$u_3 = 1.968 \times 10^{-4}$ mm

Stress

For element 1 & 2

$$\sigma_1 = E \times \frac{u_2 - u_1}{l_1} = 2\times10^5 \times \frac{(1.888\times10^{-4} - 0)}{200}$$

$$= \underline{0.188 \text{ N/mm}^2}$$

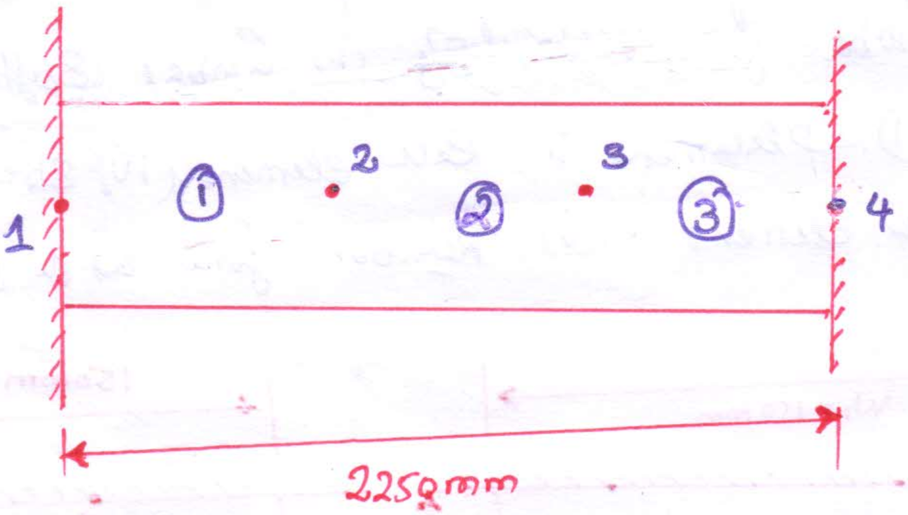
$$\sigma_2 = E \times \frac{u_3 - u_2}{l_2}$$

$$= 2\times10^5 \frac{(1.968\times10^{-4} - 1.888\times10^{-4})}{200.}$$

$$\sigma_2 = \underline{0.008 \text{ N/mm}^2}.$$

HW 3

The three bar assembley is shown in fig (i). A force of 2500N is applied in the x direction at node 2. The length of each elements is 750mm. Take $E = 4\times10^5 \, N/mm^2$ and $A = 600 \, mm^2$ for elements 1 & 2. Take $E = 2\times10^5 \, N/mm^2$ and $A = 1200 \, mm^2$ for element 3. Nodes 1 & 4 are fixed.



Calculate the following.

(i) Global Stiffness matrix

(ii) Displacements of nodes 2 & 3

(iii) Reactions at nodes 1 & 2

$$[K] = 1\times10^5 \begin{bmatrix} 3.2 & -3.2 & 0 & 0 \\ -3.2 & 6.4 & -3.2 & 0 \\ 0 & -3.2 & 6.4 & -3.2 \\ 0 & 0 & -3.2 & 3.2 \end{bmatrix}$$

$u_1 = u_4 = 0$

$u_2 = 5.127\times10^{-3} mm$

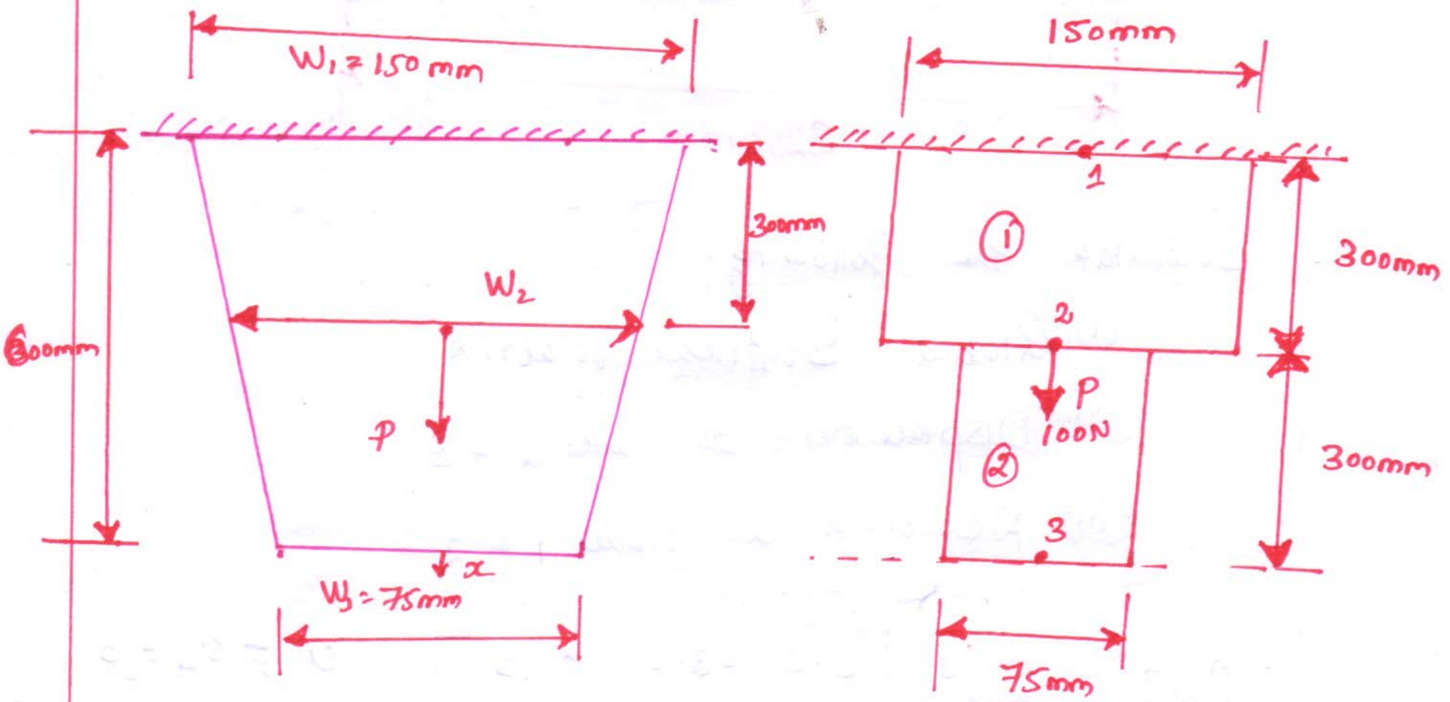
$u_3 = 2.604\times10^{-3} mm$

$R_1 = -1640 N$, $R_2 = -52N$, $R_3 = 25N$, $R_4 = -833 N$

(9) Consider a taper steel plate of uniform thickness $t = 25mm$ as shown in the figure. The youngs modulus of the plate, $E = 2 \times 10^5 N/mm^2$ and weight density $\rho = 8.2 \times 10^{-4} N/mm^3$. In addition to its self weight, the plate is subjected to a point load $P = 100 N$ at its mid point. Calculate the following by modelling the plate with 2FE.

(i) Global force vector $\{F\}$, (ii) Global stiffness matrix $[K]$

(iii) Displacement in each element. (iv) Stresses in each element. (V) Reaction force at the support.



Area at node 1, $A_1 = w_1 \times t_1 = 150 \times 25 = 3750 mm^2$

" " 2, $A_2 = W_2 \times t_2 = \left(\frac{w_1 + W_3}{2}\right) \times t_2 = \left(\frac{150+75}{2}\right) \times 25$

$A_2 = 2812.5 mm^2$

" " 3, $A_3 = w_3 \times t_1 = 75 \times 25 = 1875 mm^2$

Average area of element 1 $\bar{A}_1 = \frac{A_1 + A_2}{2} = \frac{3750 + 2812.5}{2}$

$$\boxed{\bar{A}_1 = 3281.25 mm^2}$$

Average area of element 2 $\bar{A}_2 = \frac{A_2 + A_3}{2} = \frac{2812.5 + 1875}{2}$

$$\boxed{\bar{A}_2 = 2343.75}$$

Body force $\{F\} = \frac{\rho \bar{A}_1 l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

Here $\bar{A}_1$ & $\bar{A}_2$ is the area of elements

for element 1 (node 1 & 2)

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \frac{0.82 \times 10^{-4} \times 3281.25 \times 300}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \begin{Bmatrix} 40.359 \\ 40.359 \end{Bmatrix}$$

for element 2 (node 2 & 3)

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{0.82 \times 10^{-4} \times 2343.75 \times 300}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= \begin{Bmatrix} 28.828 \\ 28.828 \end{Bmatrix}$$

Assemble body for $\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 40.359 \\ 69.187 \\ 28.828 \end{Bmatrix}$

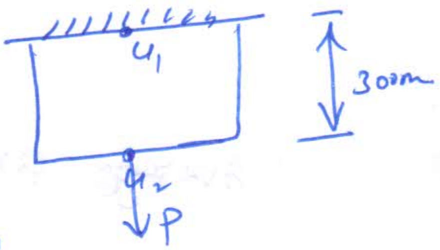
Total force

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 40.359 \\ 69.187 + 100 \\ 28.828 \end{Bmatrix}$$

$$= \begin{Bmatrix} 40.359 \\ 169.187 \\ 28.828 \end{Bmatrix}$$

FEE (1).

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\bar{A}_1 E}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

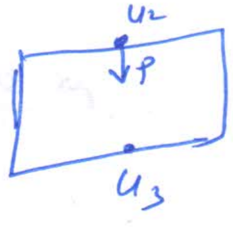


$$= \frac{328.125 \times 2 \times 10^5}{300} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= 2 \times 10^5 \begin{bmatrix} 10.937 & -10.937 \\ -10.937 & 10.937 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Element (2)

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{\bar{A}_2 E}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$



$$= 2 \times 10^5 \begin{bmatrix} 7.8125 & -7.8125 \\ -7.8125 & 7.8125 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

Assembly

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = 2 \times 10^5 \begin{bmatrix} 10.937 & -10.937 & 0 \\ -10.937 & 18.749 & -7.8125 \\ 0 & -7.8125 & 7.8125 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

Applyin boundr Condibons: $u_1 = 0$, $\cancel{F_1}$, $u_2\ u_3$ $\cancel{F_1}$, $F_2\ F_3$ equals.

$$\begin{Bmatrix} 40.359 \\ 169.187 \\ 28.828 \end{Bmatrix} = 2\times10^5 \begin{bmatrix} 10.937 & -10.937 & 0 \\ -10.937 & 18.747 & -7.8125 \\ 0 & -7.8125 & 7.8125 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$2\times10^5\,(18.747\,u_2 - 7.8125\,u_3) = 169.187$$

$$2\times10^5\,(-7.8125\,u_2 + 7.8125\,u_3) = 28.828$$

$$u_2 = 9.053\times10^{-5}\,mm \quad , u_1 = 0$$
$$u_3 = 10.898\times10^{-5}\,mm$$

Stress

$$\sigma_1 = E \times \frac{u_2 - u_1}{l_1} = \frac{2\times10^5\,(9.053\times10^{-5} - 0)}{300} = 0.060\,N/mm^2$$

$$\sigma_2 = E\frac{(u_3 - u_2)}{l_2} = 2\times10^5\frac{(10.898\times10^{-5} - 9.053\times10^{-5})}{300}$$

$$\sigma_2 = 0.0123\,N/mm^2$$

Reaction force $\{R\} = [K]\{u^*\} - \{F\}$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 2\times10^5 \begin{bmatrix} 10.937 & -10.937 & 0 \\ -10.937 & 18.749 & -7.8125 \\ 0 & -7.8125 & 7.8125 \end{bmatrix} \begin{Bmatrix} 0 \\ 9.053\times10^{-5} \\ 10.898\times10^{-5} \end{Bmatrix} - \begin{Bmatrix} 40.357 \\ 169.187 \\ 28.828 \end{Bmatrix}$$

$$= \begin{Bmatrix} -198.02 \\ 169.187 \\ 28.828 \end{Bmatrix} - \begin{Bmatrix} 40.359 \\ 169.189 \\ 28.828 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -238.379 \\ 0 \\ 0 \end{Bmatrix}$$

(P1)

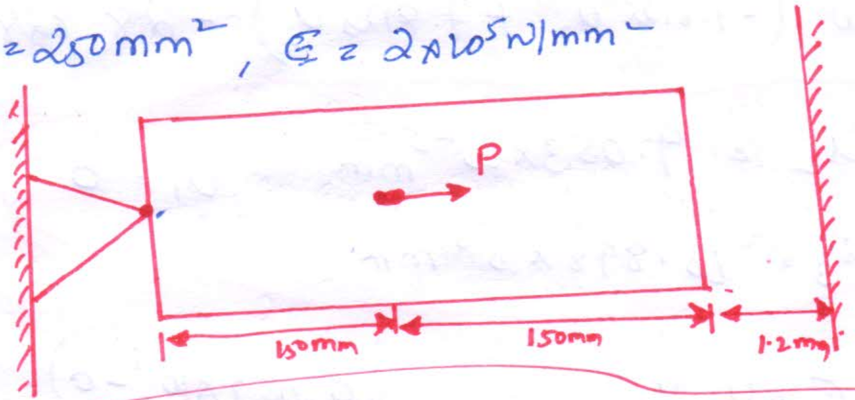
HW (4) A rod subjected to an axial load P = 600kN is applied as shown in figure. Divide the domain into two elements. Determine the following.

(a) Displacement at each node.

(b) Stresses in each element

(c) Reactions at each node point.

Take $A = 250\,mm^2$, $E = 2\times10^5\,N/mm^2$



$$dL = \frac{PL}{AE} = \frac{600\times10^3\times150}{250\times2\times10^5}$$

$$= 1.8\,mm$$

Node 3 can have the displacement 'u_3' only upto 1.2mm. after that contact with wall, so $u_3 = 1.2\,mm$.

Result:

$u_1 = 0$ $\sigma_1 = 2000\,N/mm^2$ (tension)

$u_2 = 1.5\,mm$ $\sigma_2 = -400\,N/mm$ (compress)

$u_3 = 1.2\,mm$

$R_1 = -500\,kN$

$R_2 = 0\,kN$

$R_3 = -100\,N$.

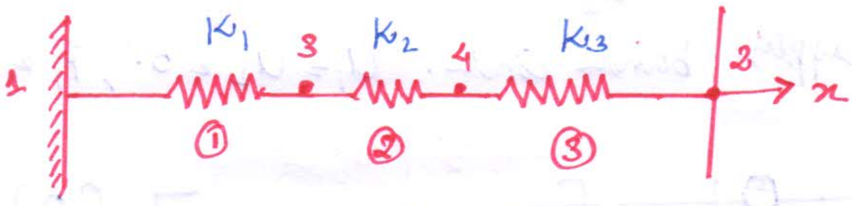
A Spring assemblage with arbitarly numberen nodes are shown in figure. The nodes 1 & 2 are fixed and a force of 500kN is applied at node 4 in the x direction. Calculate the following.

(i) Global stiffness matrix

(ii) Nodal displacements.

(iii) Reactions at each nodal point.

Take spring constant $K_1 = 100\,kN/m$, $K_2 = 200\,kN/m$, $K_3 = 300\,kN/m$



$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = K \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

element ①, node (1 & 3)

$$\begin{Bmatrix} F_1 \\ F_3 \end{Bmatrix} = K_1 \begin{array}{c} \begin{array}{cc} a_{11} & a_{13} \end{array} \\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} \begin{Bmatrix} u_1 \\ u_3 \end{Bmatrix}$$

$$= \begin{array}{c} \begin{array}{cc} a_{11} & a_{13} \end{array} \\ \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \end{array} \begin{Bmatrix} u_1 \\ u_3 \end{Bmatrix}$$

Element ② node ③ & ④

$$\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = K_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{array}{c} \begin{array}{cc} a_{33} & a_{34} \end{array} \\ \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \end{array} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$

element 3 node (3 + 2)

$$\begin{Bmatrix} F_4 \\ F_2 \end{Bmatrix} = \begin{array}{cc} a_{44} & a_{42} \\ \left[\begin{matrix} 300 & -300 \\ -300 & 300 \end{matrix}\right. & \left.\right] \end{array} \begin{Bmatrix} u_4 \\ u_2 \end{Bmatrix}$$

$$\begin{bmatrix} a_{44} = 300 & a_{42} = -300 \\ a_{24} = -300 & a_{22} = 300 \end{bmatrix}$$

Assembly

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} a_{11} = 100 & a_{12} = 0 & a_{13} = -100 & a_{14} = 0 \\ a_{21} = 0 & a_{22} = 300 & a_{23} = 0 & a_{24} = -300 \\ a_{31} = -100 & a_{32} = 0 & a_{33} = 300 & a_{34} = -200 \\ a_{41} = 0 & a_{42} = -300 & a_{43} = -200 & a_{44} = 500 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

apply bound condn. $u_1 = u_2 = 0$. , $F_1 = F_2 = F_3 = 0$

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 100 & 0 & -100 & 0 \\ 0 & 300 & 0 & -300 \\ -100 & 0 & 300 & -200 \\ 0 & -300 & -200 & 500 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 300 & -200 \\ -200 & 500 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$ resolving the Eqn

$u_3 = 0.9091$ m

$u_4 = 1.364$ m.

Reaction force $\{R\} = [K]\{u^*\} - \{F\}$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = \begin{Bmatrix} -90.91 \\ -409.20 \\ 0 \\ 0 \end{Bmatrix}.$$